

THE VIRTUAL KNOT CONCORDANCE GROUP IS NOT ABELIAN

MICAH CHRISMAN

The concordance classes of knots in the 3-sphere form an abelian group \mathcal{C} under the connected sum operation. Boden and Nagel proved that \mathcal{C} embeds as a subgroup into the center of the long virtual knot concordance group \mathcal{VC} . While the structure of the classical knot concordance group has been extensively studied in both the smooth and topological categories, little is known about the structure of \mathcal{VC} . Virtual knots can be realized as knots in thickened surfaces $\Sigma \times I$, where Σ is closed and oriented. As not all knots in $\Sigma \times I$ are homologically trivial, the usual tools of classical knot concordance (e.g. signature functions, the Arf invariant, and the algebraic concordance group), cannot be directly generalized to all virtual knots. Here we construct some concordance invariants of virtual knots that are instead closely related to Milnor's concordance invariants of classical multi-component links. The invariants are derived from the lower central series of an extension of the group of a virtual knot. Using these extended Milnor invariants, we will give new examples of non-slice virtual knots having trivial writhe polynomial, generalized Alexander polynomial, graded genus, Rasmussen invariant, and parity projection. Furthermore, we will show that in contrast to the classical knot concordance group, the virtual knot concordance group is not abelian.

DEPARTMENT OF MATHEMATICS, THE OHIO STATE UNIVERSITY, MARION, OHIO
E-mail address: `chrisman.76@osu.edu`